A study of Unobserved Components in Forward Foreign Exchange rates; The MIP

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Abstract

Is an autoregressive moving average model for the unobserved forward risk premium component always identifiable? Is the signal extraction-based approach always feasible? In this paper, we point out a theoretical framework to shed the light on the statistical problem of model identification. We find out that whenever a model for the unobservable forward risk premium is unidentifiable, we identify a new class of functions that we call: the noise generating functions (Hereafter NGF) . These functions circumvent the model identification problem and help us make insights on the noise variances. We demonstrate that a model for the risk premium in the forward exchange rate is not always identifiable and the signal extraction methodology is not always feasible. In addition, our theoretical statements are applied to the empirically evidenced models within the related literature.

Keywords: autoregressive moving average, forward risk premium, signal extraction, noise, forward exchange rate

1. Introduction

Not only do forward risk premia evoke much debate but also they are central to the theory in/of international finance. To make insights on modeling the forward risk premium component in the forward exchange rate is sine qua non for understanding its behavior and predicting it. In addition, the enlightenment of the labyrinthine clues of the forward premium puzzle (Note 1) makes easier the implementation of adequate policy tools for Central Banks. A huge body of the empirical literature has been documenting many forward risk premium component modeling approaches. In fact, continuous time vs discrete time models, linear vs nonlinear models, parametric vs nonparametric models, observed vs unobserved factor models, and regression-based vs signal extraction-based models have been performed. Engel (1996) surveyed several techniques of modeling and testing forward risk premium features such that the consumption capital asset pricing model, the latent variable model and portfolio-balance models. Diko, Lawford and Limpens (2006) investigated the presence of electricity forward risk premia in a continuous time framework and using an unobserved factor model. They adopted nonlinear and nonparametric estimation techniques. Bernoth, Von Hagen and De Vries (2010) performed an unobserved factor model to futures exchange rates. Fama (1984) implemented a regression-based approach where the forward risk premium is unobserved but it is explained via observed variables, however Wolff (1987) set up the signal extraction-based approach wherein the forward risk premium is modeled as an unobserved component. Also Cheung (1993) modeled risk premia in forward exchange rates as unobservables and pointed out a signal extraction modeling strategy. Bhar and Chiarella (2009) compared the signal extraction approach in continuous-time settings and discrete-time settings of forward risk premia. Rezessy (2010) applied three approaches from which the signal extraction approach and made a crosscheck based on them. Moreover, Cavaglia, Verschoor and Wolff (1994) pointed out, using a survey forecast data, a direct measurement of the forward risk premium and so it becomes observable. On the other hand, Bidarkota (2004) found out that the signal plus noise model failed to isolate statistically significant risk premium components from the noise. Furthermore, Jacobs (1982), Boyer and Adams (1988) and Bekaert and Hodrick (1993) shed the light on the measurement errors, model misspecifications and errors-in-variables problems whenever the regression-based approach is carried out. Gospodinov (2009) argued that the widely reported empirical literature of regressing the future exchange return on the current forward premium evidences several econometric limits that should be alleviated. In addition, Djeutem (2013) stated that the forward premium puzzle, in a context wherein agents doubt the specification of their models, is explained by a model misspecification.

Within the growing body of the empirical literature, when the financial researcher decides to handle with the forward premium anomaly, he has to choose between modeling the forward risk premium components as observables or unobservables. The former case is veracious if and only if we observe the conditional expectation of the future spot exchange rate. Indeed, Nijman, Palm and Wolff (1993) pointed out two conditions that must be satisfied to make the forward risk premium be an observed component. Once these two conditions are satisfied, the forward risk premium becomes observable and equals the forward premium. These conditions are: first, the spot exchange rate pursues a random walk stochastic process (Note 2) then the conditional appreciation/depreciation is equal to zero. Second, the semi-strong market efficiency (event studies) coincides with the weak-form market efficiency (tests for return forecastability) (Note 3). It is obvious that the required assumptions to get the forward risk premium observed are strongly restrictive. Thereby we dismiss this case. When we decide to model the forward exchange risk premium as unobservable, we have to choose between explaining it via observed variables and directly modeling it as unobservable. To study the time variation in premia and other features, we can set up either a regression-based modeling strategy or a signal-extraction modeling strategy. As mentioned by Wolff (1987), the regression-based approach has shortcomings and depends on the researcher's choice of dependent (endogenous) and independent (exogenous) variables. This arbitrariness is also dictated by the availability of the underlying data. On the other hand, the signal extraction-based approach circumvents the problem of arbitrariness of the traders'information set and it models the risk premium component as a whole, at the expense of identifying an exact econometric model for the signal. Thus, we alleviate the problem of modeling either the systematic risk or the individual relative risk aversion as a constant or a time-varying parameter.

The pertaining related literature, which considers the forward risk premium as unobservable, does not precise whether the hypothesized model for the unobserved forward risk premium component is identifiable or not and does not analyse the case wherein the model is unidentifiable. Furthermore, the previous empirical literature did not emphasize the statistical problem of model identification. In this paper, we aim to answer the following questions: Is an autoregressive moving average (Hereafter ARMA) model for the unobserved forward risk premium component always identifiable? If not, what are the underlying implications?

The remainder of the paper is organized as follows. Section 2 sets up the signal extraction preliminaries. Section 3 points out general autoregressive (AR) and moving average (MA) order conditions for unobserved stochastic processes. Section 4 implements a theoretical framework. Section 5 apllies theoretical issues to previously evidenced models in the literature. Section 6 concludes.

2. The Signal Extraction Preliminaries

The signal extraction methodology emanates from the engineering branch. It consists of writing a model in a state-space form (SSF) and applying the Kalman Filter (KF). A SSF deals with two equations: a measurement equation and a transition or state equation. The measurement equation is also called the signal plus noise model. It involves an observed time-series as a sum of two unobserved components: the signal and the noise. The signal or the unobserved state variable is deemed as buried in the noisy environment. It is a kind of interference between the signal and the noise. Thus the former should be extracted from the latter. To do so, a state equation which describes the signal stochastic process has to be specified and a filtering algorithm has to be run. The KF aims to isolate the unobservable signal from the unobservable noise. The signal extraction methodology is as follws: (a) to identify an ARMA model for the observable time-series, (b) to make an assumption on the noise stochastic ARMA process, (c) to infer an ARMA model for the unobservable signal using Ansley, Spivey and Wrobleski (Hereafter ASW) (1977)'s summation theorem (Note 4) of moving average processes, (d) to derive a SSF, and (e) to set up the KF.

2.1 The Signal plus Noise Model

Our starting point is the Fama (1984)'s definition of the forward exchange rate: the forward foreign exchange rate at time t for a delivery at time t+1 is the market determined certainty equivalent (Note 5). Therefore, it is the sum of the Markowitz forward risk premium, at time t, and the conditional expectation, made at time t, of the future spot exchange rate at time t+1. It follows:

5.2 The NGFand Their Implications

For each previously cited empirical model, we highlight its corresponding NGF. The following Table 4 reports all observed models mentioned in Table 3 and gives for each one , M^*_P, $_{NGF}$, and hypothetical model(s) causing overdetermination.

Table 4. Observable models and their corresponding

Observable model for ,, , ,	: The set of hypothetical models for the signal		M^*_P: The identifiable model for the signal	$_{NGF}$: The set of hypothetical models for the signal giving rise to NGF			Hypothetical Model causing overdetermination
MA 1,1	1, /1	,1	1	1,	/	1	
MA 1,2	1, /2	,1		1,	/	2	
MA 2,1	2, /1	,2	MA 2,1	2,	/	2	2
MA 2,2	2, /2	,2	MA 2,1	2,	/	2	2
1	1, /0	,1	1	1,	/	1	
2	2, /0	,2	MA 2,1	2,	/	2	2
MA 1	/1	,0			/	1	
MA 2	/2	,0			/	2	

We will concentrate on three categories of ARMA models: mixed ARMA processes, pure AR processes, and pure MA processes for the observable time series, , , , , . From each category, we will deal with two models. The rationale behind this choice is that each category will deal with two models leading to two systems of equations: a system with two equations and three unknowns and a system with three equations and four unknowns. Moreover, our choice encompasses the most evidenced models for the observed time series.

Empirically, the NGF will differ from a sample to another given that for each sample we get different values of the parameters , and . Although the NGF differ from a sample to another, they have to get the same fundamental characteristics" as 'positiveness, convergence and bijection. In fact, the convergence of the NGF is a sine qua non condition for the forward foreign exchange market partial equilibrium. Namely, if the first source of noise, , is infinite, the demand as well as the supply function will be null and therefore the market mechanism will be' truncated.' Side by side, if the second source of noise, , does not converge, the variance of the forward risk premium component will consequently diverge, and so does' 'the variance of the demand as well as the variance of the supply. In spite the unidentified noise variances, we have identified upper and lower bounds that do not depend on the unknown parameter , We have identified boudaries in a general framework and the NGF as well as the unconditional... d forward MA 1,1 observed variance will converge. Furthermore, the noise variances are of opposite" variation. A plausible question arises from the fact that the NGF covary in opposite directions: how do the signal variance and the first source of noise variance covary? It is easily shown that they negatively covary. From an empirical point of view, if one is willing to vary the unknown MA coefficient in order to minimize the signal variance, he should take into account that the noise variance, , will rise and' vice versa. Indeed, the signal variance and the second source of noise variance, , covary" in the same direction with respect to the unknown MA coefficient . It is essential to focus on' the ' signal-to-noise ratio, the signal variance over the first source of noise variance, and ' to deepen insights on. In fact, if the underlying ratio is greater than one then more (less) than half of the time variation in forward exchange rates is explained by the time variation in forward risk premia components (by the time variation in noise). Otherwise, less (more) than half of the time variation in forward exchange rates is explained by the time variation in forward risk premia components (by the time variation in noise). The signal-to-noise ratio is of most importance within the signal extraction analysis. It sheds the light on the explanation of the time variation in the context of a signal plus noise model. This can be helpful when one is willing to simulate the unknown MA coefficient . Another important ratio in the signal extraction-based approach is the second source of noise variance over the signal' variance. It conveys us the information whether the time variation in the signal emanates essentially from the random components or not. In fact, if the ratio is greater than one half then more than half of the time variation in the signal is of a random nature. Otherwise, more than or exactly half of the time variation in the forward risk premia components emanates essentially from the systematic components.

The same reasoning, as in section 5.2.2 and 5.2.4, is applied for the positiveness of the NGF, the convergence, the determination of upper and lower bounds and the significantly useful signal-to-noise ratios.

6. Conclusion

Not for nothing do we pinpoint the underlying topic. Our paper proposes a synthesis of previously theoretical as well as empirical research and calls attention to a crucial problem, which is identifying an ARMA model for the unobserved forward risk premia. This paper focuses on the statistical problem of model identification for the unobservable forward risk premium component. We set up a theoretical framework to study the model identification problem for the signal. In fact, the true ARMA model for the unobserved signal could be either identifiable or unidentifiable. Indeed, the signal extraction approach is feasible only for the identifiable class of ARMA models for the forward risk premium signal buried in the noise. Otherwise, it is infeasible. Whenever an ARMA model for the signal is unidentifiable, we identify a new class of functions that we call: the NGF. They are noise variances expressed as functions of the unknown forward risk premium MA coefficients, they are bijective and they are upwardly and downwardly bounded. We mathematically show that an identifiable model for the forward risk premium component does not always exist and the signal extraction approach is not always feasible. As a matter of fact, the true model is not always the identifiable one. Thus, it is well-founded to deepen insight on the class of unidentifiable models which are described by the NGF. To apply our theoretical findings, we consider the empirically evidenced ARMA models within the related literature and we analyse each case pointing out its corresponding NGF.

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